



RESEARCH DEPARTMENT

**A 'stationary-phase' method for
shaping the vertical radiation patterns
of u.h.f. transmitting aerials**

RESEARCH REPORT No. RA-6

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**THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION**

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PATTERNS OF U.H.F. TRANSMITTING AERIALS**

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Section	Title	Page
	SUMMARY	1
1.	INTRODUCTION	1
2.	RADIATION PATTERN SYNTHESIS BY THE STATIONARY-PHASE METHOD	2
	2.1. The Synthesis Equation	2
	2.2. General Solution of the Synthesis Equation	3
	2.3. Constant-amplitude Solution for a Cosecant Pattern	4
	2.4. Application to Null-filling	5
3.	APPLICATION TO U.H.F. TRANSMITTING AERIALS	6
	3.1. General	6
	3.2. Panel Aerials	8
	3.3. Comparison with other Methods	10
4.	CONCLUSIONS	10
5.	REFERENCES	10
6.	APPENDIX : Derivation of the Stationary-Phase Solution for the Radiation-Pattern Integral .	11

A 'STATIONARY-PHASE' METHOD FOR SHAPING THE VERTICAL RADIATION PATTERNS OF U.H.F. TRANSMITTING AERIALS

SUMMARY

The radiation-pattern integral is approximated by Kelvin's method of stationary-phase and the solution is employed to synthesize a cosecant radiation pattern from a constant-amplitude distribution. The method of synthesis is applied to the problem of null-filling the vertical radiation patterns of practical ultra-high-frequency transmitting aerials and the results are briefly compared with those obtainable by other methods.

PRINCIPAL SYMBOLS

$A(x)$ amplitude distribution
 d source separation (wavelengths)
 $\hat{E}$ amplitude of main beam of radiation pattern
 $I(x)$ current distribution along linear source
 $(= A(x)\exp\{j\phi(x)\})$
 k amplitude of radiation pattern (defined in Equation (2))
 m null-filling index
 m_q effective null-filling index of the q^{th} minimum
 $(= \pi q P_q)$
 M point along source at which $A(x)$ changes amplitude
 N length of radiating aperture (wavelengths)
 $p(\sin \theta)$ radiation pattern function

P_q amplitude of radiation pattern at the q^{th} minimum
 q number of minimum in v.r.p.
 r number of source in array
 s parameter (defined in Equation (8))
 x distance along linear source (wavelengths)
 α value of x at point of "stationary-phase"
 θ angle below horizontal
 θ_0 beam-tilt angle
 (θ_1, θ_2) angular range of synthesis
 λ wavelength
 $\phi(x)$ phase distribution
 ϕ_i phase distribution of isotropic sources
 ϕ_r phase distribution of panels
 ψ variable of integration $(= x - \alpha)$

1. INTRODUCTION

An ideal broadcast service provides uniform average field strength at all distances up to the limit of the service area. At ultra-high-frequencies (u.h.f.) a reasonable approximation to this ideal can be achieved by making the vertical radiation pattern (v.r.p.) of the transmitting aerial proportional to cosec θ where θ is the declination angle from the horizontal. In practice the main beam of the v.r.p. is tilted downwards by a small angle θ_0 , corresponding to the limit of the service area or horizon (whichever is nearer), in order to reduce the power wasted by radiation in unwanted directions. A typical specification for the radiation pattern $|p(\theta)|$ is written as,

$$|p(\theta)| \propto \frac{m\hat{E}}{\pi N} \text{cosec}(\theta - \theta_0) \quad (1)$$

where $\hat{E}$ is the amplitude of the main beam, N the length of the radiating aperture in wavelengths (λ), and m the so-called null-filling index*.

The significance of the index m is best understood by recalling that the radiation pattern of a uniformly illuminated aperture, $\sin(N\pi \sin \theta)/\pi \sin \theta$,

has a side-lobe envelope given by $(1/\pi) \text{cosec} \theta$; the envelope corresponds to a null-filling index $m = 1.0$. This is illustrated in Fig. 1 which shows idealised beam-tilted v.r.p.s with null-filling below the horizontal corresponding to $m = 0.50$ (50%) and $m = 1.0$ (100%); the dotted curve is the v.r.p. of the equivalent uniformly fed array. In practice, the v.r.p. should satisfy Equation (1) over the angular range $\theta - \theta_0 = \sin^{-1} 1/N$ (the first null-point) up to typically $\theta \approx \pi/6$.

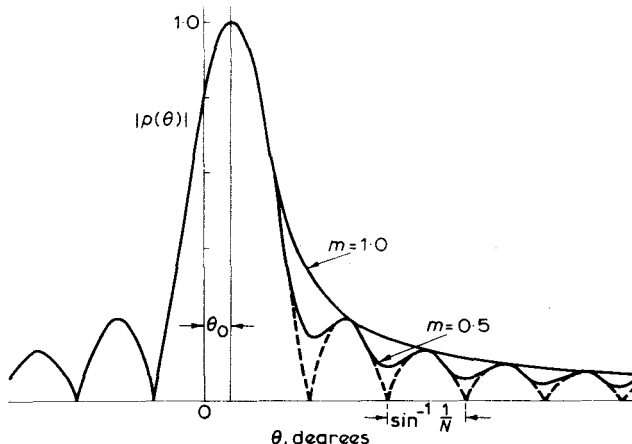


Fig. 1 - Idealised null-filled vertical radiation patterns for an $N\lambda$ array with beam-tilt θ_0

* 100m is often referred to as the 'percentage null-filling'.

From a practical standpoint the simplest method of approximating to such a radiation pattern is to use an arrangement of sources in which equally spaced tiers carry currents of equal amplitude and specified phase. Such an arrangement can be realized quite simply by energizing the aerial elements through branch-feeders whose lengths correspond to the prescribed phase distribution.

Some methods for deriving the required phase distributions with such an arrangement have been given in a previous report¹ but the particular solutions obtained suffer the disadvantage that the null-filling index m cannot normally exceed about 50%. In the method to be described, patterns can be synthesized in which the effective null-filling range is extended to about 100%.

The radiation pattern from an aperture can be represented by an integral function of the fields existing in the plane of the aperture; this radiation-pattern integral is usually expressed as a Fourier transform of the aperture distribution. In synthesis, the problem is to determine the distribution that gives a specified pattern and this requires the solution of an integral equation. Fourier inversion of the radiation-pattern integral cannot be employed in general because the phase of the pattern is arbitrary; also there may be difficulty in restricting the distribution to the dimensions of the aperture. The new null-filling method is based on a "stationary-phase" approximation² of the radiation-pattern integral and it therefore gives a solution which is consistent with the laws of geometrical-optics. It follows that the new method is restricted to aeriels whose radiating lengths are large compared with the wavelength. The attraction of the stationary-phase solution from a synthesis standpoint is that it gives expressions for amplitude and phase distributions in terms of the power pattern, thus obviating the need to specify the phase of the far-field radiation pattern; moreover, restricting the distribution to the aperture presents no analytical difficulty.

The application of the stationary-phase method to radiation pattern synthesis was first pointed out by Dunbar³ and later used by Shanks⁴ to synthesize a variety of shaped patterns from a linear array whose phase distribution is specified. In the present application, the amplitude distribution is specified (constant over the radiating length) and the phase distribution determined for a cosecant-squared power pattern; the discrete source currents for the practical array are evaluated by appropriately sampling the synthesized continuous distribution. The report commences with a simplified derivation of the stationary-phase solution of the radiation-pattern integral and shows how this solution provides the required phase distribution. The result is then applied to a 32λ array in which the sources are grouped into co-phased panels of lengths 2λ and 4λ , including aeriels with panels of mixed lengths. The computed radiation patterns are

compared with one another and also with those obtainable from other methods.

2. RADIATION PATTERN SYNTHESIS BY THE STATIONARY-PHASE METHOD

A "synthesis equation" is derived here from energy considerations using the principle of stationary-phase; a more rigorous derivation of the solution of the radiation-pattern integral is given in the Appendix (Section 6). The synthesis equation will be solved for the phase distribution required to give a cosecant radiation pattern from a constant-amplitude array.

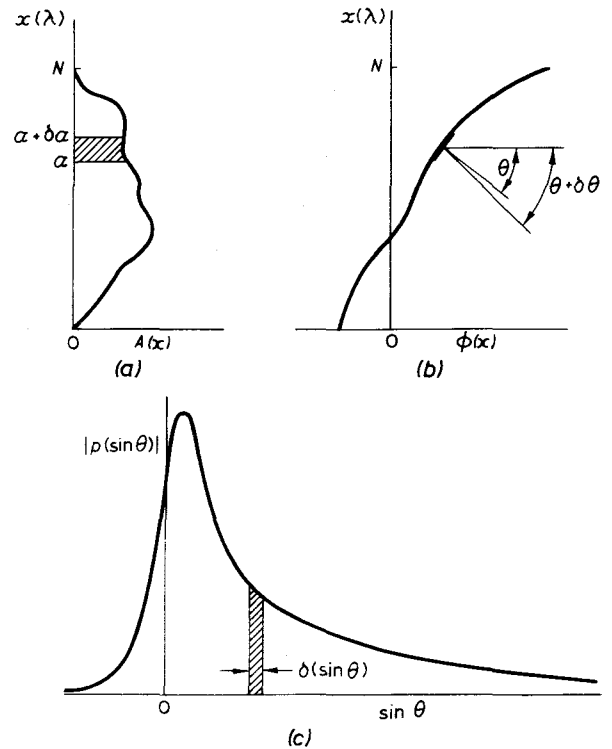


Fig. 2 - A linear source of N wavelengths with current distribution $I(x) = A(x)\exp\{j\phi(x)\}$

(a) amplitude distribution (b) phase distribution
(c) radiation pattern

2.1. The Synthesis Equation

Let us consider a linear source with a current distribution $I(x) = A(x)\exp\{j\phi(x)\}$ along its length ($N\lambda$); the functions $A(x)$ and $\phi(x)$ are the amplitude and phase distributions respectively shown in Figures 2(a) and 2(b). The complex far-field radiation pattern of the source $p(\sin\theta)$ is given by the following radiation-pattern integral,

$$p(\sin\theta) = \frac{1}{k} \int_0^N A(x) \exp\{j\phi(x) - 2\pi x \sin\theta\} dx \quad (2)$$

where θ is the inclination below the horizontal, x the position co-ordinate along the source in wave-

lengths and k the magnitude of the radiation pattern (the significance of this constant will appear later, in Section 2.2).

Suppose now that $x = \alpha$ is a point of *stationary-phase* for a particular value of θ , i.e. at $x = \alpha$ we have,

$$\frac{d}{dx}\{\phi(x) - 2\pi x \sin\theta\} = 0$$

Thus at $x = \alpha$,

$$\frac{d}{dx}\phi(x) = 2\pi \sin\theta \quad (3)$$

When N is large, the integrand of Equation (2) will be highly oscillatory except near the point $x = \alpha$, where the phase is stationary*. The major contribution to the integral, therefore, comes from an incremental aperture length δx in the immediate vicinity of the point $x = \alpha$.

We now proceed heuristically to derive a synthesis equation using an argument involving energy considerations which is based on the type of approximation employed in geometrical optics. It is hoped that this will give an insight into the physical nature of the mathematical approximations involved; a formal mathematical derivation of the equation is given in the Appendix (Section 6).

It has already been pointed out that for a given value of θ the major contribution to the radiation-pattern integral comes from that part of the integrand which is close to the point of stationary-phase; the declination ($\sin\theta$) corresponding to this point is given by Equation (3). It is now assumed that each increment of aperture δx is a radiating source which directs practically all its power over a narrow angular range $\delta(\sin\theta)$ which is centred on the stationary-phase declination (Equation (3)). This assumption is consistent with the fact that the *total* real powers in the aperture and the radiation pattern must be identical. Accordingly we equate the power in an incremental length δx of the source to the power in an increment $\delta(\sin\theta)$ of the radiation pattern (Fig. 2(c)) at the stationary-phase declination; this relation is

$$k^2 |p(\sin\theta)|^2 \delta(\sin\theta) \simeq A^2(\alpha) \delta x \quad (4)$$

where $A(\alpha)$ is the amplitude of the current at the stationary-phase point. The relationship between the increment of aperture δx and the corresponding increment of $\sin\theta$ may be derived from Equation (3) as follows,

$$\frac{\delta(\sin\theta)}{\delta x} \simeq \frac{d}{dx}(\sin\theta) = \frac{1}{2\pi} \phi''(\alpha)$$

* The point $x = \alpha$ is assumed to be single-valued.

where $\phi''(\alpha)$ denotes the second derivative of $\phi(x)$ with respect to x at $x = \alpha$. Combining this result with Equation (4) we have,

$$k^2 |p(\sin\theta)|^2 \phi''(\alpha) \simeq 2\pi A^2(\alpha) \quad (5)$$

Used in conjunction with Equation (3), Equation (5) is the basis of pattern synthesis by the stationary-phase method. Inserting $\sin\theta$ from Equation (3) gives the final result,

$$k^2 |p\{\phi'(\alpha)/2\pi\}|^2 \phi''(\alpha) \simeq 2\pi A^2(\alpha) \quad (6)$$

which is a "synthesis equation" relating the distribution functions $A(\alpha)$ and $\phi(\alpha)$ through the power pattern $|p(\sin\theta)|^2$. For any given pattern function, there is in general no unique solution for the amplitude and phase distributions. However, if the phase distribution $\phi(\alpha)$ is specified, Equation (6) provides an explicit expression for the amplitude distribution $A(\alpha)$; if, on the other hand, we specify $A(\alpha)$, Equation (6) becomes a non-linear differential equation for the phase distribution $\phi(\alpha)$. In general, either case can provide a reasonably accurate solution to synthesis problems provided that the conditions of stationary-phase are met; these will normally be satisfied for sources with large length-to-wavelength ratios.

In the present application, the function $A(\alpha)$ is taken to be a constant over the length of the source ($0 \leq x \leq N$) and zero elsewhere; this corresponds to the constant-amplitude condition. Before proceeding with this specific case we shall first consider the general solution for synthesis given by Equation (6) for those cases in which the amplitude distribution is arbitrarily specified. The existence of such a solution shows that the stationary-phase method can be used in principle to solve a wide variety of pattern-shaping problems in addition to the null-filling case to be considered here.

2.2. General Solution of the Synthesis Equation

It will be convenient to write the synthesis relation given by Equation (6) in the form,

$$k^2 |p(s)|^2 s' \simeq A^2(\alpha) \quad (7)$$

$$\text{where} \quad s = \phi'(\alpha)/2\pi = \sin\theta \quad (8)$$

and $s' = ds/d\alpha$. The function $|p(s)|^2$ represents the shape of the power pattern and k^2 is its magnitude. From energy considerations we see that synthesis of an arbitrary pattern shape must normally be restricted to a finite angular range $\theta_1 \leq \theta \leq \theta_2$ and in fact it will be shown that k is directly determined by θ_1 and θ_2 . Separating variables in Equation (7) and integrating gives,

$$k^2 \int |p(s)|^2 ds \simeq \int A^2(\alpha) d\alpha + C_1 \quad (9)$$

where C_1 is a constant of integration; from this equation we can obtain the function s , i.e.

$$s = s(\alpha, k, C_1)$$

Integration of Equation (8) now gives the solution for the phase distribution $\phi(\alpha)$ as,

$$\phi(\alpha) \simeq 2\pi \int s(\alpha, k, C_1) d\alpha + C_2 \quad (10)$$

where C_2 is another constant of integration. Equation (10) is applicable for those cases in which $s(\alpha, k, C_1)$ is a single-valued explicit function of α .

The pattern amplitude k and the constant of integration C_1 are given by the end-points of the synthesis range (θ_1, θ_2) through the stationary-phase relation (Equation (8)), viz

$$\begin{aligned} s(0, k, C_1) &= \sin \theta_1 \\ s(N, k, C_1) &= \sin \theta_2 \end{aligned} \quad (11)$$

Equations (11) are solved simultaneously to yield k and C_1 . The boundary conditions can, of course, be inverted by interchanging θ_1 and θ_2 in Equations (11) so that the largest phase-slope occurs at the bottom ($\alpha = 0$) rather than at the top ($\alpha = N$) of the source. Finally, the second constant of integration C_2 is determined by the position of the phase reference point α_0 for the source currents, i.e. inserting $\phi(\alpha_0) = 0$ into Equation (10) gives,

$$C_2 = -2\pi \int s(\alpha_0, k, C_1) d\alpha \quad (12)$$

It is normally convenient to place α_0 at one end of the source so that α_0 is equal to either 0 or N in Equation (12).

In applying the above solution to distributions of constant amplitude, we simply substitute a constant value of $A(\alpha)$ into Equation (9). An account of the general applicability of the method to the synthesis of arbitrary radiation patterns is not straightforward and is outside the scope of this report. It can be stated at this point, however, that the method does give satisfactory synthesis of cosecant and rectangular shaped radiation patterns from constant-amplitude distributions.

The particular case of synthesis for a cosecant pattern from a linear source of constant amplitude and its application to the v.r.p. null-filling problem will now be considered in detail.

2.3. Constant Amplitude Solution for a Cosecant Pattern

The radiation pattern we require to synthesize has been given by Equation (1). At this stage, the beam-tilt θ_0 will be omitted (i.e. setting $\theta_0 = 0$) in order to simplify the analysis; the required tilt-

angle can always be realized after synthesis, by superimposing a linear phase progression onto the synthesized distribution. The pattern function to be inserted into Equation (9) is therefore

$$p(\sin \theta) = \text{cosec } \theta = 1/s \quad (13)$$

which gives $|p(s)|^2 = s^{-2}$. The constant-amplitude condition is satisfied by making $A(\alpha) = 1$ for $0 \leq \alpha \leq N$ and $A(\alpha) = 0$ for $-\infty < \alpha < 0$ and $N < \alpha < \infty$. The differential synthesis equation (Equation (9)) then gives

$$s(\alpha, k, C_1) \simeq -k^2/(\alpha + C_1)$$

so that the phase distribution, from Equation (10), is given by

$$\phi(\alpha) \simeq -2\pi k^2 \log(\alpha + C_1) + C_2$$

The boundary conditions of Equations (11) and (12) provide the relations

$$C_1 = -k^2 \text{cosec } \theta_1 = -(k^2 \text{cosec } \theta_2 + N)$$

and

$$C_2 = 2\pi k^2 \log C_1$$

with the phase-reference point for the source taken at the base of the aperture, $\alpha_0 = 0$. The final expression for the phase distribution $\phi(x)$ is, therefore

$$\phi(x) \simeq -2\pi k^2 \log(1 - \frac{\sin \theta_1}{k^2} x) \quad (14)$$

$$\text{with } k^2 = N(\text{cosec } \theta_1 - \text{cosec } \theta_2)^{-1} \quad (15)$$

The parameter α has been replaced by x since the solution is valid for all points $x = \alpha$. Equation (14) gives the phase distribution required for a linear source of constant amplitude and length $N\lambda$ to provide a radiation pattern $k \text{cosec } \theta$ over an angular range $\theta_1 \leq \theta \leq \theta_2$ ($\theta_1 > 0$). The solution is valid for sources whose radiating lengths contain a large number of wavelengths (N large) and the accuracy of the synthesized patterns will increase the larger N is made.

The radiation pattern is unchanged (in amplitude) if the phase distribution given by Equation (14) is reversed in sign and inverted; for some applications, the alternative distribution $-\phi(N - x)$ may be more convenient to realize in practice since the largest phase-slope then occurs at the lower end rather than at the upper end of the aerial. Choice of θ_1 , the lower limit of the synthesis range, is important since with $\theta_1 \gg \theta_2$ it determines the amplitude k of the cosecant pattern through Equation (15) and hence also the effective degree of null-filling. The upper limit of the range, θ_2 , determines the largest phase-slope at the end of the aerial through the relation $\phi'(N) = 2\pi \sin \theta_2$; this must also be chosen carefully since with practical

arrays (sampled distributions) it becomes difficult to realize large phase-slopes unless closely spaced elements are employed.

The above solution can easily be extended to cases where there exists non-uniform power division over the length of the aerial. For example, let us suppose that the aerial is divided into two unequal sections of constant amplitude with $A(x) = A_1$ for $0 < x < M$ and $A(x) = A_2$ for $M < x < N$. The lower section is used to synthesize the pattern over the range $\theta_1 \leq \theta < \theta_{12}$ and the upper section over the range $\theta_{12} < \theta \leq \theta_2$ where θ_{12} has to be determined. The required expressions become particularly simple if the total radiated power is set equal to that radiated by the same aerial uniformly fed with $A(x) = 1$; we then have $MA_1 + (N-M)A_2 = N$. The amplitude of the power pattern k^2 is then given by Equation (15) as before and it can be shown that the angle θ_{12} at which the patterns due to the two sections meet is given by,

$$N \operatorname{cosec} \theta_{12} = MA_1^2 \operatorname{cosec} \theta_2 + (N-M)A_2^2 \operatorname{cosec} \theta_1 \quad (16)$$

provided that the phase distribution is made continuous over $x = M$. The phase distributions $\phi_1(x)$ and $\phi_2(x)$ over the lower and upper sections of the aerial can now be written directly, from Equation (14) as,

$$\phi_1(x) \simeq -\frac{2\pi k^2}{A_1^2} \log \left(1 - \frac{A_1^2 \sin \theta_1}{k^2} x \right) \quad [0 < x < M] \quad (17)$$

and

$$\phi_2(x) \simeq \phi_1(M) - \frac{2\pi k^2}{A_2^2} \log \left\{ 1 - \frac{A_2^2 \sin \theta_{12}}{k^2} (x-M) \right\} \quad [M < x < N] \quad (18)$$

It is easy to show that, for the uniformly fed aerial with $A_1 = A_2 = 1$ Equations (17) and (18) degenerate into Equation (14) by substituting Equations (15) and (16).

2.4. Application to Null-Filling

In applying the synthesis technique to the problem of null-filling v.r.p.s it is convenient to consider a cosecant pattern with zero beam-tilt, i.e. with $\theta_0 = 0$ in the pattern specification (Equation (1)). The null-filled pattern can always be tilted through any desired angle by subsequently superimposing a linear phase progression on to the synthesized current distribution. It should be pointed out, however, that the nature of synthesis is such as to produce of itself some incidental beam-tilt with the null-filled v.r.p. and this would normally have to be taken into account in choosing the final phase progression.

As an example of the technique, a phase distribution $\phi(x)$ was determined (through Equation (14)) in order to give a cosecant radiation pattern from a 20λ source ($N = 20$) over the angular range $\theta_1 = \sin^{-1}(1/2N)$ to $\theta_2 = \pi/6$; the lower limit is approximately half the inclination of the first null-point in the pattern of the corresponding uniformly fed source. Equation (15) gives the desired pattern amplitude over the stated range as $k = 0.725$. The discrete phase distribution ϕ_r for the equivalent array of 40 isotropic sources spaced 0.5λ apart was obtained by directly sampling the continuous distribution $\phi(x)$,

$$\text{i.e. } \phi_r = \phi(r-0.5)$$

The resulting radiation pattern is shown in Fig. 3 together with the distribution $\phi(x)$ (inset). It will

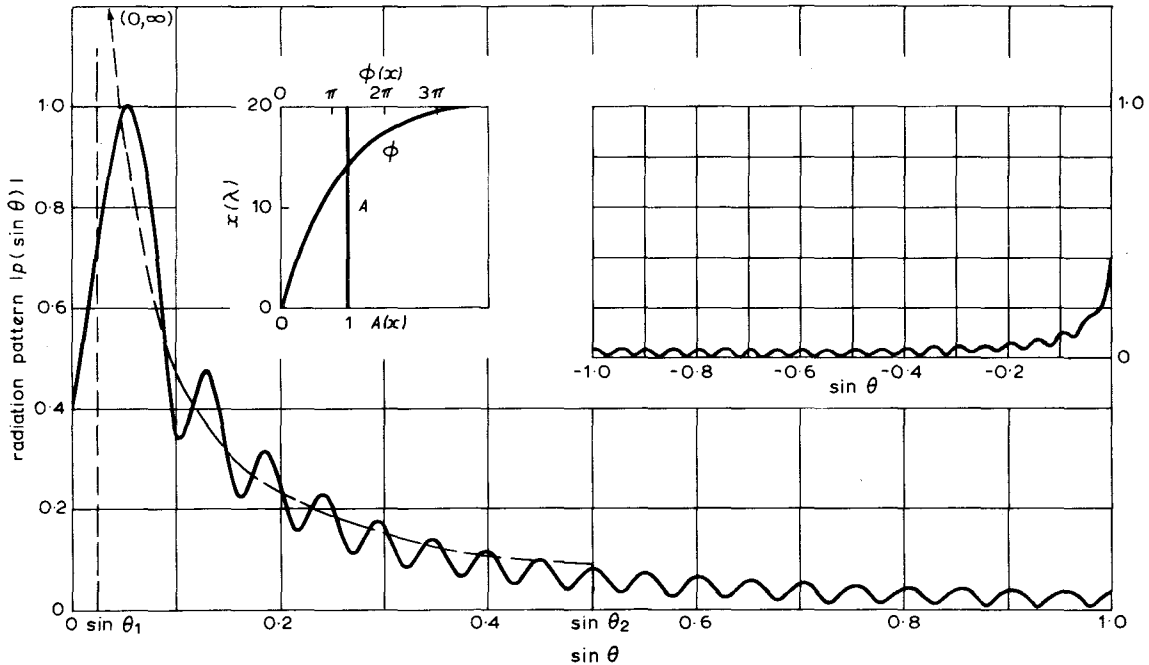


Fig. 3 - Synthesis of a cosecant radiation pattern from a 20λ array : 40 isotropic sources spaced 0.5λ apart with $k = 0.725$
 ———— desired pattern, $k \operatorname{cosec} \theta$, $\theta_1 \leq \theta \leq \theta_2$

be seen that the synthesized pattern oscillates about the desired cosecant pattern (dashed curve) with the largest errors occurring near to the end points, θ_1 and θ_2 ; near θ_1 , the main beam presents a poor approximation and towards θ_2 the oscillatory pattern begins to fall below the desired level. The beam-tilt in the synthesized pattern is a function of the lower angular limit θ_1 selected and its presence makes the "null-filling" of the shifted pattern non-uniform. Nevertheless, the null-filling is reasonably smooth and is acceptable without further perturbation of the phases of the individual source currents.

The case of a non-uniformly fed source has also been investigated in order to determine whether the stationary-phase method could be extended to such cases. The above 20λ source was assumed to be split into two sections of equal length, the upper section being energized with twice the power of the lower section. The phase distributions over the sections required for cosecant pattern synthesis were therefore determined from Equations (17) and (18) with $M = 10$, $A_1 = 0.816$, and $A_2 = \sqrt{2}A_1$. The resulting v.r.p. was a reasonably smooth null-filled pattern which differed insignificantly from that obtained in the uniformly-fed case.

With normalized radiation patterns* the effective degree of null-filling m_q of the q^{th} null may be determined from Equation (1) by the simple expression,

$$m_q = \pi q P_q \quad (19)$$

where P_q is the normalized amplitude $|p(\sin\theta)|_{\text{norm}}$ of the q^{th} minimum in the synthesized pattern. In the example given in Fig. 3 the result of setting $k = 0.725$ over the range $1/2N < \sin\theta < 1/2$ is as follows:

- (i) m_q rises from just above 100% for the first minimum ($q = 1$) to a maximum of approximately 140% for $q = 3, 4$ and 5 , falling to 100% for the final minimum in the synthesis range ($q = 8$).
- (ii) outside this range, subsequent minima decrease in amplitude as q increases with finally $m_{18} \approx 50\%$.
- (iii) the gain reduction due to the departure of the phase distribution from the co-phased condition is approximately 2.0 dB.
- (iv) the maximum ratio of adjacent maxima and minima of the pattern is approximately 6 dB over the side-lobe range $1 \leq q \leq 8$.

It should be noted that the null-filling index is a measure of the extent to which the v.r.p. minima

* The amplitude of the main beam is set to unity, i.e. $E = 1.0$.

approach the envelope of the normalized, beam-tilted pattern of a uniform array ($m = 1$ curve, Fig. 1); this should not be confused with the $k\text{cosec}\theta$ curve shown in Fig. 3. Thus when m exceeds 100% the minima lie above the envelope of the uniform-array pattern and the maxima are even higher. The increased power radiated in the sidelobes accounts for the gain reduction.

The ratio of adjacent maxima and minima of a null-filled pattern is an important parameter. Patterns with large ratios tend to vary rapidly with frequency and, in addition, they provide a service rather susceptible to propagation effects. In these respects, a pattern is normally taken to be acceptable provided that this ratio does not exceed 8 dB. In the example given in Fig. 3, the ratio is approximately 2 dB less than this figure (para. (iv) above) and the pattern would therefore be acceptable.

The overall degree of null-filling can be adjusted by varying k (which incidentally adjusts the beam-tilt also); for a fixed upper limit θ_2 this means shifting θ_1 (Equation (15)). Since m_q is nearly proportional to k over a large range of q , in those cases where $\sin\theta_2 \gg \sin\theta_1$, we have the useful result,

$$m_q \propto (\sin\theta_1)^{1/2} \quad [\theta_1 \ll \theta_2] \quad (20)$$

In general, the most important minimum of the pattern is the first one which appears below the main beam ($q = 1$) and null-filling adjustment would normally be carried out at that point.

3. APPLICATION TO U.H.F. TRANSMITTING AERIALS

The method of synthesis is now applied to the problem of null-filling the vertical radiation patterns of practical u.h.f. transmitting aerials. There is no loss of generality in restricting the discussion to aerials whose radiating lengths are 32λ and whose first pattern minima are to be null-filled by not less than 75% (i.e. $m_1 \geq 0.75$).

3.1. General

Two types of u.h.f. aerial will be considered. The first is the low-power type which normally employs equally spaced tiers of simple vertical dipoles; the second is the high-power type which normally consists of identical groups (or panels) of equally spaced horizontal dipoles.

The cosecant pattern synthesis given in Section 2.4 was repeated for a 32λ array of isotropic sources spaced 1.0λ apart with a synthesis range of $1/2N < \sin\theta < 1/2$, i.e. $k = 0.718$ from Equation (15). The null-filling, again larger than 100%, was rather excessive for the present application and so the

level of the first minimum m_1 was lowered to about 75% using Equation (20) to obtain a new lower limit θ_1 , for the cosecant synthesis. The new range was $0.0089 < \sin\theta < 0.50$ and the resulting sampled phase distribution ϕ_r and corresponding v.r.p. are shown in Fig. 4; this result can be used as the basis for the design of u.h.f. transmitting aerials whose radiating lengths are 32λ .

In one type of aerial commonly employed*, the isotropic sources of the previous array are replaced by vertical half-wave dipoles, so that if the distribution ϕ_r of Fig. 4 is used, the v.r.p. which is obtained from the phased array of dipoles is given

* At BBC-2 relay stations.

TABLE 1
Effective Null-Filling Indices (%)
 $100m_q (= 100\pi q P_q)$ for 32λ Arrays

q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	gain (dB) reduction
Fig. 4 with dipole factor	80	97	98	99	95	91	87	83	78	72	63	58	53	48	50	1.33
Fig. 6 Panels 7, 8 tilted	80	100	110	100	78	90	130	56	50	47	28	26	20	16	45	1.39
Fig. 6 Panel 8 tilted	77	91	94	100	110	130	130	25	28	31	28	26	20	44	90	1.36
Fig. 7 Panels 9, 10 tilted	80	97	103	100	86	94	130	43	70	63	35	38	41	57	94	1.32
Fig. 7 Panel 10 tilted	80	97	103	88	71	75	130	50	98	94	70	76	80	88	94	1.32

by the product of the v.r.p. shown and the dipole factor $\cos\{(\pi/2)\sin\theta\}/\cos\theta$. The result of this is shown in Table 1 (first row) which gives the amplitudes of the pattern minima over the range $0 < \sin\theta < 0.5$ in terms of the percentage null-filling indices, $100m_q$. Such a result is acceptable from the null-filling standpoint although the minima between $2 < q < 6$ are a little excessive; the secondary maxima (sidelobes) are also rather high, but this is the penalty paid for achieving the large m_q 's with a constant amplitude distribution.

Beam-tilt adjustment, if required, is performed by superposing a linear phase progression onto the logarithmic distribution ϕ_r ; a beam-tilt θ_0 is obtained from a phase-slope of $2\pi d \sin\theta_0$ radians/tier up the array, where d is the element spacing in wavelengths.

It should be pointed out that the asymmetrical current distribution over the aerial due to the logarithmic nature of ϕ_r has two principal side-effects if the aerial is conventionally branch-fed and split into two identical sections, one above the other. First, the v.r.p. from each section working singly would be very different; the upper section gives a reasonably smooth v.r.p. whereas the lower section provides practically no null-filling at all. Secondly, the phase distribution ϕ_r makes the impedance of each section such that aerial impedance compensation** can be reasonably good by using a conven-

** A detailed consideration of aerial impedance with logarithmic phase distributions is outside the scope of this report.

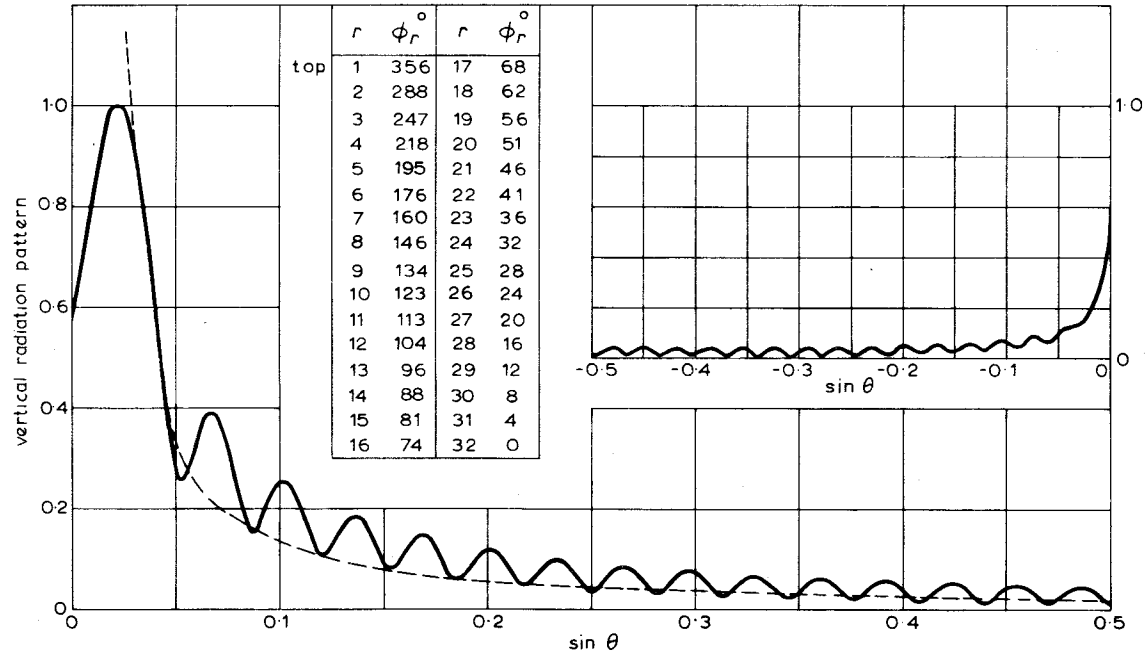


Fig. 4 - Vertical radiation pattern for 32 isotropic sources spaced 1.0λ apart with logarithmic phase distribution ϕ_r ; $k = 0.540$
--- null-filling reference, $m = 1.0$, Equation (1)

tional diplexer⁵ for power splitting; the actual degree of compensation will largely depend on the net beam-tilt employed.

3.2. Panel Aerials

At high-power u.h.f. stations with horizontal polarization, the inter-tier spacing can be reduced to around 0.5λ and it is then practicable to build up the aerial from standard vertical groups or panels of horizontal dipoles. Typical panels are $2-4\lambda$ long; a 4λ panel, for example, would carry 8 dipoles spaced 0.5λ apart and 0.25λ away from a reflecting screen where λ is the wavelength at the centre frequency. Columns of panels are mounted around the mast axis, the individual columns being fed normally with progressive phase-shift around the mast; we concern ourselves here with the v.r.p. from a single column of panels. Synthesized distributions will now be considered that are subject to the restriction that the phase distribution over every panel must be the same, since the complexity arising from special phase adjustments within each panel is to be avoided if possible.

The solution for the 32λ array given in Section 3.1 above, will be used and the phase distribution of the stack of panels Φ_r , based on the distribution ϕ_r shown in Fig. 4. In this case, however, we use the negative-inverted distribution $-\phi_{N-r+1}$ (i.e. lagging) so that the large phase-slope at the end of the distribution appears at the lower end of the aerial facing downwards, as shown in Fig. 5(b). The reason for this is that at the steep end of the phase distribution the panel(s) must normally be

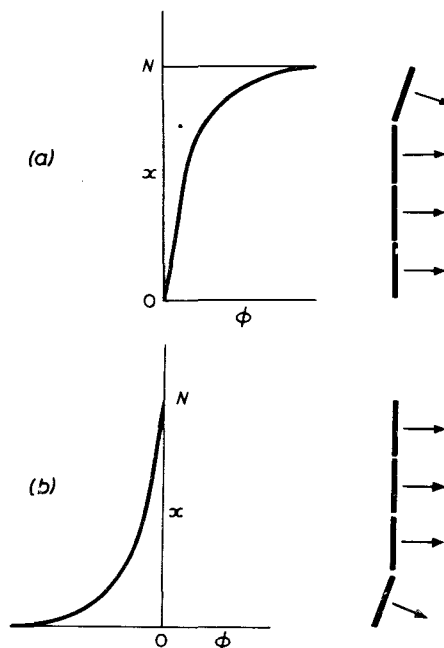


Fig. 5 - Phase distributions and corresponding panel-tilt arrangements

- (a) leading phase distribution
(b) lagging phase distribution

physically tilted, in addition to being electrically phased, in order that the phase profile be accurately followed. Panels tilted away from the main array will give less inter-panel mutual coupling than the alternative arrangement (Fig. 5(a)) and this makes realization of the synthesized distributions somewhat easier.

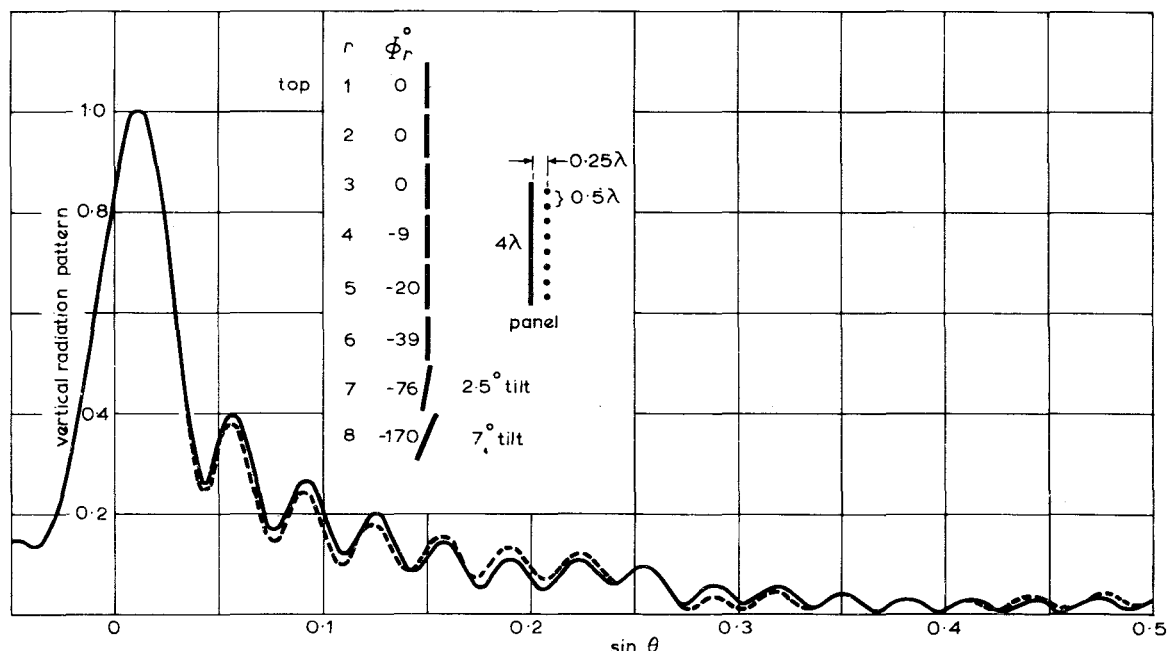


Fig. 6 - Vertical radiation patterns for a 32λ aerial with 8 panels; panels of 8 horizontal dipoles spaced 0.5λ apart and 0.25λ from reflecting screen with phase distribution Φ_r
 ——— v.r.p. for arrangement shown - - - - - panel 8 (bottom) tilted only

Starting from the sampled distribution ϕ_r (1.0λ spacing), the electrical phases of the panels Φ_r were taken as the mean of the source phases contained by the panel; for 2λ panels, $\Phi_r = (\phi_{2r} + \phi_{2r-1})/2$ and for 4λ panels, $\Phi_r = (\phi_{4r} + \phi_{4r-1} + \phi_{4r-2} + \phi_{4r-3})/4$. Panel-tilts at the bottom of the aerial were determined from the differential phase shift between the outermost sources contained by the panel; for 2λ panels, the mechanical tilt-angle is approximately $(\phi_{2r} - \phi_{2r-1})/2\pi$ radians whereas for 4λ panels the tilt-angle is normally taken to be $(\phi_{4r} - \phi_{4r-3})/6\pi$ radians, which, in practice, differs insignificantly from the alternative and fuller expression $(\phi_{4r} + \phi_{4r-1} - \phi_{4r-2} - \phi_{4r-3})/8\pi$ radians. These tilt-angles are increased by about 7% in order to make some allowance for the fact that the equivalent electrical phasing produced by source-stagger is reduced by the factor $\cos\theta$ at angles inclined to the horizontal.

Some examples of the v.r.p.s produced by panel aeriels with phase distributions and panel-tilts obtained by the above procedure are shown in Figs. 6 and 7. Considering first Fig. 6, the "jump" in the v.r.p. near $\sin\theta = 0.25$ is due to the coarse quantisation of the original distribution into 4λ groups; the jump is accentuated if the panel-tilt is restricted to the lowest panel only (dotted curve). The effective null-filling in these two cases is satisfactory although the v.r.p. minima are rather large near $\sin\theta = 0.1$ (Table 1, rows 2 and 3); the loss of aerial gain due to de-phasing is about 1.4 dB. Figure 7 shows the v.r.p.s obtained from an aerial with mixed panels (6 of 4λ and 4 of 2λ). The jump

in the v.r.p. at $\sin\theta = 0.25$ has been reduced by the presence of the finer quantization of the phase profile over the lower part of the aerial. In this case there is little advantage in tilting both of the lower two 2λ panels as compared with tilting the lowest one only. Table 1 (rows 4 and 5) again gives the levels of the v.r.p. minima in terms of the effective null-filling indices m_q ; loss of gain due to de-phasing is slightly less than before at 1.3 dB. The overall v.r.p. is satisfactory from a null-filling standpoint.

In the above examples we have considered aeriels containing an even number of panels. In these cases, a conventional split-feeder system gives uniform power division over the aerial. If, on the other hand, the aerial is built up from an odd number of identical panels and split into two equally powered sections, the amplitude distribution will become non-uniform; in this case Equations (17) and (18) are used for v.r.p. synthesis.

We conclude from the above results that there appears to be little advantage in employing the mixed-panel design as against the more economical design involving 4λ panels only, where the stationary-phase solution is used. Finally, it is worth mentioning that, in practice, it may be more convenient to tilt a panel about its end-point rather than its centre-point; provided that the electrical phase of the panel is suitably adjusted, the previous results also apply to this situation.

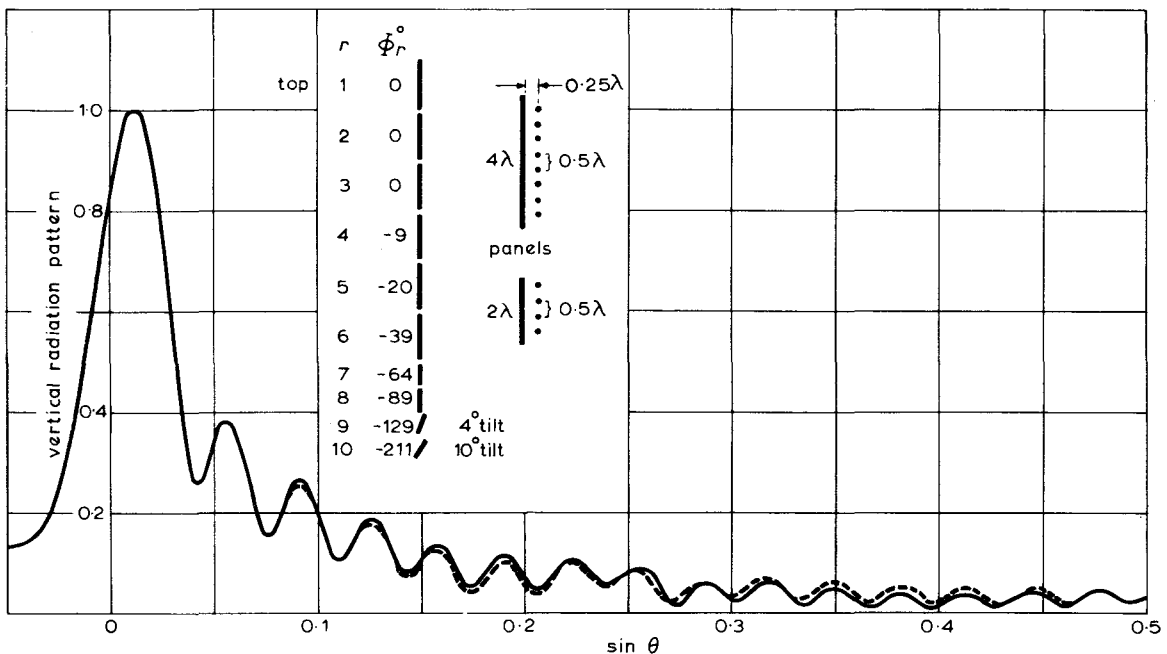


Fig. 7 - Vertical radiation patterns for a 32λ aerial with six 4λ panels and four 2λ panels; panels comprise horizontal dipoles spaced 0.5λ apart and 0.25λ from reflecting screen with phase distribution Φ_r
 ——— v.r.p. for arrangement shown - - - - - panel 10 (bottom) tilted only

3.3. Comparison with Other Methods

The method of radiation pattern synthesis employed here for null-filling the v.r.p.s of u.h.f. aerials is based on a constant-amplitude current distribution; such distributions are attractive from a practical standpoint because they can be realized with simple branch-feeder systems. Solutions involving both amplitude and phase tapering of the current distribution, the Woodward cosecant solution⁶ and its variants for example, become difficult to realize because of the additional complication of amplitude weighting the currents to the aerial elements. Nevertheless, solutions to the null-filling problem using amplitude tapering can provide v.r.p.s which are more accurate and smoother approximations of the cosecant pattern than those obtainable from the present method. The more interesting comparison is with an alternative constant-amplitude solution.

The phase-perturbation method (Reference 1) provides an alternative constant-amplitude technique for shaping the v.r.p.s of u.h.f. (and v.h.f.) aerials. The basic phase distributions in this solution are symmetrical and the null-filling they provide is uniform although in practice cannot normally exceed about 50%. The new method given here depends on a logarithmic asymmetrical phase distribution and can provide null-filling up to and beyond 100% if required; the null-filling is a little non-uniform, however, and secondary maxima in the v.r.p.s can attain rather high levels. One advantage of the asymmetrical solution is that less power is wasted in radiation at angles above the horizontal.

The stationary-phase method for shaping v.r.p.s is, therefore, superior to the phase-perturbation method in respect of the available null-filling it provides, but, with the higher degree of null-filling, the departure in the level of the pattern from the ideal cosecant envelope results in some loss of gain.

4. CONCLUSIONS

The radiation-pattern integral has been approximated by the method of stationary-phase and the resulting synthesis equation solved for an arbitrary radiation pattern, to give the phase distribution required for a large linear source with a specified amplitude distribution. In particular, the solution has been used for synthesizing a cosecant radiation pattern from a line source of constant amplitude. This phase distribution is a logarithmic function which depends upon the length of the source and the angular range over which the cosecant pattern is synthesized.

Computed results with a limited number of patterns for discrete arrays show that the geomet-

rical-optics type of approximation involved gives reasonably accurate results for radiating apertures down to 20λ long. Applied to the problem of shaping the v.r.p.s of u.h.f. transmitting aerials, the method has been shown to give quite acceptable null-filled patterns. The effective null-fillings in the v.r.p.s of panel aerials can be placed in the range 75 to 100% which exceeds that obtainable from the previously described phase-perturbation method. However, the larger degree of null-filling results in some non-uniformity in the levels of the pattern minima and, in some cases, rather high side-lobe levels and consequent gain reduction. It is worth mentioning that the phase distributions are generally such that good aerial impedance matching can be obtained by conventional methods. As with any constant-amplitude solution, the synthesis method described here offers the practical advantage that the current distribution may be realized by simply energizing the elements of the aerial through branch-feeders whose lengths follow the synthesized phase distribution.

The general applicability of the method for synthesizing patterns of arbitrary shapes is difficult to specify; a necessary condition is that the radiating source must be of large size-to-wavelength ratio. In any particular case, the accuracy of synthesis can always be checked by pattern computation.

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6. APPENDIX

Derivation of the Stationary-Phase Solution for the Radiation-Pattern Integral

The radiation-pattern integral has been given in Section 2.1 (Equation (2)) as

$$p(\sin\theta) = \frac{1}{k} \int_0^N A(x) \exp j\{\phi(x) - 2\pi x \sin\theta\} dx \quad (21)$$

We wish to obtain an approximate result for this integral assuming that the phase of the integrand is a rapidly varying function over the range of integration (i.e. along the length of the source). The variable of integration x is expressed in wavelengths so that this condition involves the geometrical-optics type of approximation of $\lambda \rightarrow 0$.

The phase of the integrand in Equation (21) is *stationary* for

$$\phi'(x) = 2\pi \sin\theta \quad [x = \alpha] \quad (22)$$

and we assume that the stationary point $x = \alpha$ is single-valued. Kelvin's principle of stationary-phase states that the major contribution to the value of the integral is obtained in the immediate vicinity of the point $x = \alpha$ where the phase is stationary. Using this principle we can therefore write the integral as,

$$kp(\sin\theta) \simeq \int_{\alpha-\epsilon}^{\alpha+\epsilon} A(x) \exp j\{\phi(x) - 2\pi x \sin\theta\} dx \quad (23)$$

where 2ϵ is the region over which the integral has significant value. The argument of the exponential is now expanded as a Taylor series about the point $x = \alpha$, i.e.

$$\begin{aligned} \phi(x) - 2\pi x \sin\theta &\simeq \{\phi(\alpha) - 2\pi \alpha \sin\theta\} + \\ &+ \{\phi'(\alpha) - 2\pi \sin\theta\}(x - \alpha) + \frac{1}{2} \phi''(\alpha)(x - \alpha)^2 \end{aligned}$$

where the series has been truncated at the third term (second order). But the stationary-phase condition, Equation (22), reduces the second term in this expression to zero so that we have,

$$\begin{aligned} \phi(x) - 2\pi x \sin\theta &\simeq \{\phi(\alpha) - 2\pi \alpha \sin\theta\} + \\ &+ \frac{1}{2} \phi''(\alpha)(x - \alpha)^2 \end{aligned} \quad (24)$$

If we now assume that $A(x) \simeq A(\alpha)$ over the region of integration 2ϵ , and insert Equation (24) into Equation (23), the Fourier integral becomes,

$$kp(\sin\theta) \simeq A(\alpha) \exp j\{\phi(\alpha) - 2\pi \alpha \sin\theta\} \int_{\alpha-\epsilon}^{\alpha+\epsilon} \exp j\{\phi''(\alpha)(x - \alpha)^2/2\} dx \quad (25)$$

The integral in Equation (25) may be expressed in standard form by changing the variable to $x - \alpha (= \psi)$, i.e.

$$I(\alpha) = \int_{-\epsilon}^{\epsilon} \exp j\{\phi''(\alpha)\psi^2/2\} d\psi$$

Making the usual approximations involved in applying the principle of stationary-phase, the limits of integration $\pm\epsilon$ can be replaced by $\pm\infty$; the result is a Fresnel integral, i.e.

$$\begin{aligned} I(\alpha) &\simeq \int_{-\infty}^{\infty} \exp j\{\phi''(\alpha)\psi^2/2\} d\psi \\ &= \left(\frac{2\pi}{\phi''(\alpha)} \right)^{1/2} \exp j(-3\pi/4) \end{aligned} \quad (26)$$

Equations (25) and (26) together give the result,

$$p(\sin\theta) \simeq \left(\frac{2\pi}{k^2 \phi''(\alpha)} \right)^{1/2} A(\alpha) \exp j\{\phi(\alpha) - 2\pi \alpha \sin\theta - 3\pi/4\} \quad (27)$$

Equation (27) is the complex far-field radiation pattern of a line source with current distribution $A(x) \exp\{j\phi(x)\}$ assuming that the optical approximations of stationary-phase are valid. If the phase information in Equation (27) is removed by writing the corresponding expression for the power pattern we have,

$$|p(\sin\theta)|^2 \simeq \frac{2\pi A^2(\alpha)}{k^2 \phi''(\alpha)} \quad (28)$$

which will be recognized as the result obtained in Section 2.1 (Equation (5)) from energy considerations.

